Assign 4 Combinatorics

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Name:_____   Date:_____

Combinatorics

1. In how many different ways can a 3-person committee be selected from 6?

$$6C_3 = \frac{6 \times 5 \times 4}{1 \times 2 \times 3} = 20$$

2. In a 7 horse race, Bill thinks horses 1, 4, 6 will be the top 3 horses in the race, but not necessarily in that order. If Bill is correct, how many different outcomes are possible?

$$1,4,6 \quad , \quad 2\ 3\ 5\ ,7$$
$$3! \qquad \times \qquad 4!$$
$$6 \quad \times \quad 24$$
$$= 144$$

3. A committee of 3 students is to be selected from a group of 10 to be on a committee to plan a school trip. How many different committees can be selected?

$$10C_3 = \frac{10 \times 9 \times 8}{1 \times 2 \times 3}$$
$$= 10 \times 3 \times 4 = 120$$

4. In a regular heptagon ABCDEFG, how many triangles can be constructed whose vertices are among the points A, B, C, D, E, F, and G?

7 points, EACH TRIANGLE NEEDS 3 pts.
$$7C_3 = \frac{7 \times 6 \times 5}{1 \times 2 \times 3} = \boxed{35}$$

5. How many different 3-letter "words" are possible such that the letters of each word are in alphabetical order? (For example, the "word" ADF is in alphabetical order.)

$$26C_3 = \frac{26 \times 25 \times 24}{1 \times 2 \times 3}$$
$$= 13 \times 25 \times 8$$
$$= 2600$$

6. 3 pennies are to be placed onto a 5 by 5 grid. How many ways can you place the 3 pennies such that non of them share the same row or column?
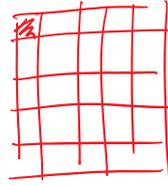
2 METHODS:

$5C_3 \times 5C_3 \times 6$

$\dfrac{5 \times 4 \times 3}{1 \times 2 \times 3} \times \dfrac{5 \times 4 \times 3}{1 \times 2 \times 3} \times 6$

$= 10 \times 60$

$= 600$

or

$\dfrac{25 \times 16 \times 9}{1 \times 2 \times 3} \leftarrow$ ORDER DOESN'T MATTER

$25 \times 8 \times 3$

$= 600$

7. An election has three different positions: President, Vice President, and Secretary. There are 4 people running for President, 3 for VP, and 5 for Secretary. If a person is voting on a ballot, how many ways can someone fill in the ballot. Assume that you can not submit an empty ballot.
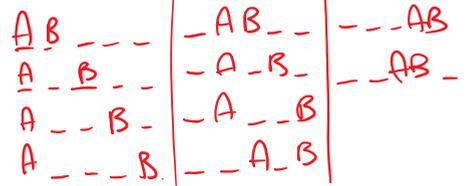
$$\text{PR.} \quad \text{VP} \quad \text{S}$$
$$4 \times 3 \times 5 = 60$$

8. How many ways can 5 people A, B, C, D and E sit in a row if A must be to the left of B but not necessarily next to each other?

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

• HALF OF THEM A left of B
• HALF OF THEM A right of B

$\left\{ \begin{array}{l} \text{or} \quad 5C_3 \times 3! \\ = \dfrac{5 \times 4 \times 3}{1 \times 2 \times 3} \times 6 \\ = 60 \end{array} \right.$

$\underline{A}\,B\;\_\_\_$
$\underline{A}\;\_\,B\;\_\_$
$\underline{A}\;\_\_\,B\;\_$
$\underline{A}\;\_\_\_\,B$

$\_\,A\,B\;\_\_$
$\_\,A\;\_\,B\;\_$
$\_\,A\;\_\_\,B$
$\_\_\,A\;\_\,B$

$\_\_\_\,A\,B$
$\_\_\,A\,B\;\_$
$\_\,A\,B\;\_$

9. How many ways can 3 boys and 3 girls sit in a row if:
   a. There are no restrictions on where they sit in the row?

$\underline{6!}$

   b. All the boys sit together, and all the girls sit together?

$\underline{3! \times 3! \times 2}$

AAA BBB
BBB AAA

   c. Only the boys must sit together, the girls choosing the other seats?

$4! \times 3!$ 　 G G G $\boxed{AAA}$

10. A committee consists of 4 men and 2 women. A subcommittee is to be formed consisting of 1 man and 1 woman. In how many different ways can the subcommittee be formed?

$$4C_1 \times 2C_1$$
$$= 4 \times 2$$
$$= 8$$

11. There are 8 boys and 12 girls in a class. A team of 5 is to be formed with 3 girls. How many different teams are possible?

$$8B \qquad 12G.$$
$$C_2 \qquad C_3$$
$$8C_2 \times 12C_3$$

$$= \frac{8 \times 7}{1 \times 2} \times \frac{12 \times 11 \times 10}{1 \times 2 \times 3}$$
$$= 28 \times 2 \times 11 \times 10$$
$$= 6160 \,//$$

12. There are 8 boys and 12 girls in a class. ~~A team of 5 is to be formed with 3 girls~~ If the teams must have at least 2 boys, how many different teams are possible?

2B3G or 3B2G or 4B1G or 5B

$$8C_2 \times 12C_3 + 8C_3 \times 12C_2 + 8C_4 \times 12C_1 + 8C_5$$

13. Three boys and their girlfriends have 6 seats at a hockey game. In how many ways can they be seated if each couple must sit together?

AB CD EF → $3! \times 2 \times 2 \times 2$
$$= 6 \times 8$$
$$= 48 \,//$$

14. There are ten teams in a school district competition. Each team plays each other team once. What is the total number of games played in the competition?

$$10C_2 = \frac{10 \times 9}{1 \times 2} = \boxed{45}$$

15. An intramural league has 4 teams. If each team is scheduled to play each of the remaining teams exactly twice during the season, how many games are scheduled

$$4C_2 \times 2$$
$$\frac{4 \times 3}{1 \times 2} \times 2 = \boxed{12}$$

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16. If there are 8 people in a room and each person shakes every other person's hand once, how many handshakes will there be?

$$8C_2 = \frac{8 \times 7}{1 \times 2}$$
$$= 28$$

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17. Nine people apply for a job in which 2 people are selected to work in an office and 3 in the work yard. In how many different ways can the selection be made?

$$9C_2 \times 7C_3$$
$$\frac{9 \times 8}{1 \times 2} \times \frac{7 \times 6 \times 5}{1 \times 2 \times 3}$$
$$36 \times 35$$
$$= 35^2 + 35$$
$$= 4225 + 35$$
$$= 4260$$

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